







# **Describing Sequential Systems**

- So far we have used Truth Tables to describe sequential systems
- Can also use Bubble Diagrams and Algorithmic State Machine Charts (ASM) to describe a sequential system.
- Another name for a sequential system is a *Finite State Machine* (FSM).
- A sequential system with N flip-Flop has 2<sup>N</sup> possible states, so the number of possible states is FINITE.

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DFF as a Finite State Machine				
A DFF is a finite state machine with two possible states. Lets call these states S0 and S1. ( <i>state enumeration</i> ).				
Furthermore, lets say when the Q output = '0', then we are in State S0, and that when Q output = '1', we are in State S1. This is called the <i>State Encoding</i> .				
$\begin{array}{c} 0 \\ \hline \\ S0 \\ q=0 \end{array} \begin{array}{c} 1 \\ \hline \\ q=1 \end{array} \begin{array}{c} S1 \\ q=1 \end{array}$				
Bubble Diagram: States represented by bubbles. State transitions represented by arrows. Labeling on arrows represent input values (in this case, the D-input!). Labeling inside bubbles represent output values.				
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#### Finite State Machine Implementation

Given an Algorithmic State Machine chart that describes a Finite State Machine, how do we implement it?????

Step #1: Decide on the State Encoding (how many Flip Flips do I use and how what should the FF outputs be for EACH state). The problem definition *may* decide the state encoding for you.

Step #2: Decide what kind of FFs to use! (We will always use DFFs in this class, but you could use JKFFs or TFFs if you wanted to).

Step #3: Write the State Transition Table.

Step #4: Write the FF input equations, and general output equations from the state transistion table.  $$_{\rm BR\,899}$$ 

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Design a *Modulo three* counter. The count sequence is:

"00"  $\rightarrow$  "01"  $\rightarrow$  "10"  $\rightarrow$  "00"  $\rightarrow$  "01"  $\rightarrow$  "10", etc.

There is an "en" input that should control counting (count when en=1, hold value when en=0). Assume ACLR line used to reset counter to "00".

How many states do we need? Well, we have three unique output values, so lets go with three states.







tate transition resent state, in	table shows next sta put values.	te, output value	es fo
Inputs(EN)	Present State	Next State	Y
0	S0	<b>S0</b>	00
0	S1	<b>S1</b>	01
0	S2	<b>S2</b>	10
1	S0	<b>S1</b>	00
1	S1	<b>S2</b>	01
1	S2	S0	10

### Decisions

- State encoding will be based on number of FFs we use.
  - Three states means the minimum number of FFs we can use two FFs  $(\log_2(3) = 2)$ .
- If we use two FFs, then could pick a state encodings like:
  - S0: 00, S1: 01, S2: 10 (binary counting order)
  - S0: 01, S1:01, S2: 11 (gray code may result in less combinational logic)
- Could also use 1 FF per state (3 FFs) and use one hot encoding
  - S0:001, S1: 010, S2: 100 (may result in less combinational logic) BR 899

# Decisions (cont.)

- What type of FF to use?
- DFF most common type, always available in programmable logic
- JKFF sometimes available, will usually result in less combinational logic (more complex FF means less combinational logic external to FF)

Lets use two FFs with state encoding S0=00, S1=01, **S2=10.** 

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Lets use **DFFs**.

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need to be in state encodin	order to get gs	to that state	. Also, us	se actual
(Inputs(EN)	Present	Next	D1D0	Y
• • •	State	State		
	(Q1Q0)	(Q1Q)*		
0	00	00	00	00
0	01	01	01	01
0	10	10	10	10
1	00	01	01	00
1	01	10	10	01
1	10	00	00	10



D-input Equations, Y equations Unoptimized equations:

D0 = EN'Q1'Q0 + ENQ1'Q0'

D1 = EN' Q1 Q0' + EN Q1' Q0

 $\mathbf{Y0} = \mathbf{Q0}$ 

Y1 = Q1

The output Y is simply the DFF outputs! Here is one case where state encoding is affected by problem definition (does not make much sense to use a different state encoding, even though we could do it). BR 8/99 15





What if we used JKFFs?						
Need to change State Transistion table to reflect JK input values.						
Inputs EN	Present State (Q1Q0)	Next State (Q1Q0)*	J1 K1	J0 K0	Y	
0	00	00	0 X	0 X	00	
0	01	01	0 X	X 0	01	
0	10	10	X 0	0 X	10	
1	00	01	0 X	1 X	00	
1	01	10	1 X	X 1	01	
1	10	00	X 1	0 X	10	
JK FF Q transitions: 0→0 (J=0, K=X); 0→1 (J=1, K=X); 1→1 (J=X, K=0); 1→0 (J=X, K=1); BR 899 17						



JK Input Equ	ations, Output Equatio	ns
Unoptimized equation	ns	
J0 = EN Q1' Q0'	K0 = EN Q1' Q0	
J1 = EN Q1' Q0	K1 = EN Q1 Q0'	
Y0 = Q0		
Y1 = Q1		
0	an simpler external optimized ecause FFs are more complex onality).	
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3 DFFs and One Hot Encoding State encoding: S0 = 001, S1 = 010, S2 = 100							
Inputs EN	Present State (O2O1O0)	Next State (020100)*	D2D1D0	Y			
0	001	001	001	00			
0	010	010	010	01			
0	100	100	100	10			
1	001	010	010	00			
1	010	100	100	01			
1	100	001	001	10			
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DFF input equations, Output Equations D0 = EN'Q0 + ENQ2 D1 = EN'Q1 + ENQ0 D2 = EN'Q2 + ENQ1Y0 = EN'Q1 + EN Q1 = Q1 Y1 = EN'Q2 + EN Q2 = Q2 In equations, because a FF Q will only be '1' in a single state, do not have to include all FFs to define state!! (Q2'Q1'Q0 = Q0!!, Q2'Q1Q0' = Q1!, Q2Q1'Q0' = Q2!!) This is one of the advantages of one-hot encoding!

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# Generic Next State Equations

Generic next state equations can be written directly from the ASM chart as an alternative to the Transition table

 $S^* =$  (conditions to remain in this state) + (conditions to enter state)

From ASM chart of modulo three counter:  $S0^* = EN'S0 + ENS2$   $S1^* = EN'S1 + ENS0$   $S2^* = EN'S2 + ENS1$ 

If One hot encoding and DFFs are used, then Generic Next State equations ARE the specific next State Equations!!

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 $\begin{array}{l} D0 = EN'Q0 + EN \ Q2 \\ D1 = EN'Q1 + EN \ Q0 \\ D2 = EN' \ Q2 + EN \ Q1 \end{array}$