

### Minimization via K-Maps

Row	A	B	C	F(A,B,C)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0

		A	
		0	1
BC	00	0	0
	01	0	0
	11	0	0
	10	1	1

$F(A,B,C) = \sum m(2,6)$   
 $= A'BC' + ABC'$   
 $= BC'(A' + A)$   
 $= BC'$

**Boolean adjacency can be used to minimize functions!**

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### Groupings on K-Maps

		A	
		0	1
BC	00	0	0
	01	0	0
	11	0	0
	10	1	1

Grouping can be read **DIRECTLY** as “BC” by looking at what is **COMMON** within the circled group.

$F(A,B,C) = BC'$

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### Example Groupings on 3-Variable K-Maps

		A	
		0	1
BC	00	1	0
	01	1	0
	11	0	0
	10	0	0

$F(A,B,C) = A'B'$

		A	
		0	1
BC	00	1	1
	01	0	0
	11	0	0
	10	1	1

$F(A,B,C) = C'$

		A	
		0	1
BC	00	1	1
	01	1	1
	11	0	0
	10	0	0

$F(A,B,C) = B'$

**Remember that top, bottom of map are adjacent!!**

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### Multiple Groupings

Want to cover all '1's with largest possible groupings.

	A	0	1
BC			
00		1	0
01		1	1
11		0	0
10		0	0

$F(A,B,C) = B'C + A'B'$

Groupings of only a single '1' are ok if larger groupings cannot be found.

	A	0	1
BC			
00		0	1
01		0	0
11		1	0
10		1	0

$F(A,B,C) = AB'C' + A'B$

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### Illegal Groupings

Illegal Grouping! Minterms are not boolean adjacent!

	A	0	1
BC			
00		1	0
01		0	1
11		0	0
10		0	0

$A'B'C'$ ,  $AB'C$  will NOT reduce to a single product term  
 $A'B'C' + AB'C = B'(A'C' + AC)$

Valid groupings will always be a power of 2. (will cover 1, 2, 4, 8, etc minterms).

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### Groupings on four Variable Maps

		AB			
		00	01	11	10
CD					
00		0	0	0	0
01		0	0	0	0
11		1	0	1	0
10		1	1	0	1

$F(A,B,C,D) = A'B'C + A'CD' + B'CD' + ABCD$

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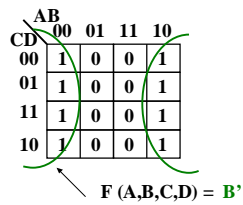
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### Other Groupings



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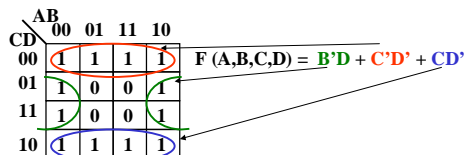
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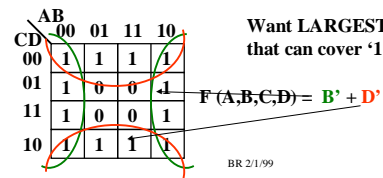
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### More than one way to group.....



Want LARGEST groupings that can cover '1's.



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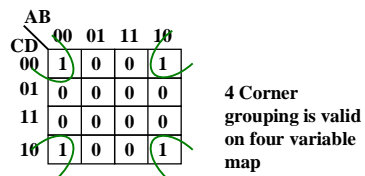
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### Four Corner Grouping on 4-Variable Map



4 Corner grouping is valid on four variable map

$$F(A,B,C,D) = B'D'$$

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### Some Definitions

BC

	0	1
00	0	1
01	0	0
11	1	1
10	1	1

AB'C' (blue circle), BC (green circle), B (red circle)

**Implicant:** Any single 1 or any group of 1's is called an implicant of F. Any possible grouping of '1's is an **implicant**.

BC

	0	1
00	0	1
01	0	0
11	1	1
10	1	1

AC' (green circle), B (red circle)

**Prime Implicant:** A covering that cannot be combined with some other covering to eliminate a variable.

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### Minimum SOPs

The minimum SOP expression consists of some (**but not necessarily all**) of the prime implicants of a function.

If a SOP expression contains a term which is NOT a prime implicant, then it **CANNOT** be minimum.

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### Prime Implicants

AB

	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	1	0	0	0
10	0	0	0	0

BC' (blue circle), A'C'D (green circle), A'B'D (red circle)

**EACH** of the coverings is a **PRIME IMPLICANT**.

Minimum SOP will have some or all of these prime implicants. The included prime implicants must cover all of the ONEs.

$F(A,B,C,D) = BC' + A'B'D$  (minimum # of PIs)  
 $= BC' + A'B'D + A'C'D$  (valid, but not minimum)  
 $\neq A'B'D + A'C'D$  (both PI's, but all '1's not included!)

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## Non-Essential vs. Essential Prime Implicants

AB \ CD	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	1	0	0	0
10	0	0	0	0

EACH of the coverings is a **PRIME IMPLICANT**.

$BC'$  ,  $A'C'D$  ,  $A'B'D$

$$F(A,B,C,D) = BC' + A'B'D \quad (\text{minimum \# of PIs})$$

Prime Implicant  $A'C'D$  is a **NON-ESSENTIAL** prime implicant because its '1's are covered by other PIs. A PI is **ESSENTIAL** if it covers a MINTERM that cannot be covered by any other PI.

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## An example with more than one solution

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	1	1	1
10	0	0	0	0

EACH of the coverings is a **PRIME IMPLICANT**.

$A'C'$

$ACD$

$A'BD$

$BCD$

Recall that a covering is a Prime Implicant if it cannot be combined with another covering to eliminate a variable.

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## Two Solutions

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	1	1	1
10	0	0	0	0

EACH solution is equally valid.

$$F(A,B,C,D) = A'C' + ACD + A'BD$$

Essential PIs

Non-Essential PIs

$$F(A,B,C,D) = A'C' + ACD + BCD$$

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## Minimal Solution

A minimal SOP will consist of **prime implicants**.

A minimal SOP equation will have all of the **essential** prime implicants on the map. By definition, these cover a minterm that may not be covered by some other prime implicant.

The minimal SOP equation may or may not include **non-essential** prime implicants. It will include non-essential prime implicants if there are '1's remaining that have not been covered by an essential prime implicant.

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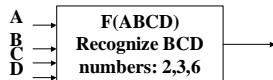
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## Don't Cares

Row	A	B	C	D	F(A,B,C,D)
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	x
11	1	0	1	1	x
12	1	1	0	0	x
13	1	1	0	1	x
14	1	1	1	0	x
15	1	1	1	1	x

Recall that Don't Cares are labeled as 'X's in truth table. Can treat X's as either '0's or '1's



Non BCD numbers are don't cares because will never be applied as inputs.

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## Don't Cares treated as '0's or '1's

AB \ CD	00	01	11	10
00	0	0	X	0
01	0	0	X	0
11	1	0	X	X
10	1	1	X	X

Treat X's as 1's when can get a larger grouping. All X's do not have to be covered.

$$F(A,B,C,D) = CD' + B'C$$

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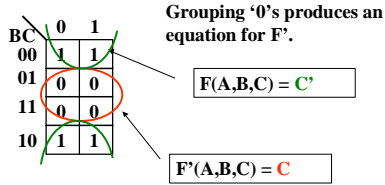
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### Minimizing '0's



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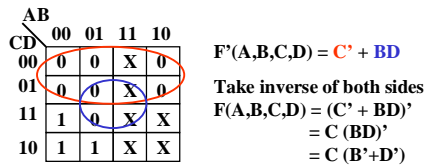
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### Minimize 0's, then Complement to get POS



Minimizing zeros, then applying inverse to both sides is a way to get to minimum POS form!!!!

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### What do you need to know?

- How to minimize functions using 2,3,4 variable Kmaps.
  - Group 1's to get to minimal SOP form
  - Group 0's then take complement to get to minimal POS form.
- Definitions of implicant, prime implicant, non-essential prime implicant, essential prime-implicant.
  - Be able to recognize these on a K-map.
- How to treat 'X's on a K-map.

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